



INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

## *On Traffic Light Control of Regular Towns*

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*apport  
de recherche*





## On Traffic Light Control of Regular Towns

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**Abstract:** We present a hierarchical way of designing the traffic light control of regular towns. We first give a model for light synchronization based on Petri nets and minplus algebra. Using this modelling we decompose the problem in three parts : – computation of the cycle length of each traffic light, – computation of the starting time of each traffic light cycle, – computation of the proportion of the green and red length in a cycle. The example of the Bahía Blanca, city of Argentina, is given.

**Key-words:** transportation, traffic light control, Petri net, maxplus algebra

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## **Sur la commande des feux de circulation d'une ville régulière**

**Résumé :** On présente une méthode hiérarchique pour déterminer les plans de feux d'une ville de forme régulière. On présente d'abord un modèle pour la synchronisation des feux basé sur les réseaux de Pétri et l'algèbre minplus. Utilisant cette modélisation on décompose le problème en trois niveaux : – le calcul du temps de cycle d'un feu, – le calcul du temps de démarrage des cycles de chaque feu, – le calcul de la proportion de rouge et de vert de chaque cycle. L'exemple d'une ville, à la géométrie régulière, Bahía Blanca, en Argentine, est donné.

**Mots-clés :** transport, commande des feux de circulation, réseau de Pétri, algèbre maxplus

## 1 Introduction.

We describe a way to compute a traffic light plan in regular towns.

In a traffic light plan three quantities must be computed for each traffic light : – the cycle length of lights, – the starting time of the light cycle (phase), – the proportion of red and green in a cycle.

We call “regular” a town which possesses some symmetry at least approximately. Here we consider the example of the center of Bahía Blanca a map of which is given in Figure 5. It is approximately invariant by 2 translations, the first is North-South of length 2 blocks, the second is East-West also of length 2 blocks.

For such a symmetric town we build a Petri net describing the synchronization between all the lights of the town based on virtual car circulation at a given speed and a simple description of the flows of these cars.

By analysing this model we show a decomposition between the flow evaluation and the time spent in the system.

To optimize the time spent in a system by a car we have to design “green waves”. If we correctly choose the light cycle length, we show that it is possible to design four systems of compatible green waves which assure that we can join two points in the town, at prescribe speed, meeting at most one red light. This result is valid only when there is no saturation that is when the flows in all the streets are smaller than the virtual car flows.

The maximal virtual car flows are given by saturation of the slowest resources which are the junctions in our case. The saturation is reached when the junctions are always occupied.

In practise we propose to adapt the cycle length proportion to the real flow and to maintain the coordination between the lights given by the four systems of green waves.

The assumption of geometric regularity of the town is not so restrictive. It can be achieved more often by adapting the speed on each portion of street in such a way that the times needed to cover each block stay equal.

## 2 Max-Plus modelling of Petri nets.

Let us explain the way to compute the throughput of a quite general class of Petri nets which can be interpreted in term of stochastic control. For more details see [1].

**Definition 1** A continuous Petri net is defined by

$$\mathcal{N} = (\mathcal{P}, \mathcal{Q}, M, \rho, m, \tau) ,$$

where :

1.  $\mathcal{P}$  is a finite set whose elements are called places ;
2.  $\mathcal{Q}$  is a finite set whose elements are called transitions ;
3.  $M \in (\mathbb{R}^+)^{\mathcal{P} \times \mathcal{Q} \cup \mathcal{Q} \times \mathcal{P}}$  are the arc multipliers that is  $M_{pq}$  (respectively  $M_{qp}$ ) denotes the number of arcs from transition  $q$  to place  $p$  (respectively from place  $p$  to transition  $q$ );

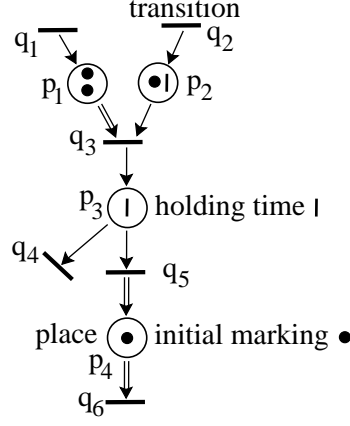


Figure 1: A Petri net.

4.  $\rho : \mathcal{Q} \times \mathcal{P} \rightarrow \mathbb{R}^+$  verifying :

$$\sum_{q \in p^{\text{out}}} \rho_{qp} = 1, \quad \forall p \in \mathcal{P},$$

is the routing policy which gives the imposed proportion of fluid going from place  $p$  to transition  $q$  with respect to quantity of fluid entering place  $p$  ;

5.  $m \in (\mathbb{R}^+)^{\mathcal{P}}$  is the initial marking, that is :  $m_p$  is the amount of fluid available at place  $p$  at starting time ;
6.  $\tau \in (\mathbb{R}^+)^{\mathcal{P}}$  is the holding time which is the time that a molecule of fluid has to stay in place  $p$  before leaving.

The dynamics of the system are determined by the *firing* of transitions. A transition fires as soon as there is enough amount of fluid, as required by the routing policy, available in all places  $p \in q^{\text{in}}$  upstream<sup>1</sup> the transition  $q$  (by “available” we mean that the fluid spent the minimum holding time in its place). The total quantity consumed by  $q$  at place  $p$  at time  $t$  is  $Z_q(t)M_{qp}$  (where the amount of firing at  $q$  is denoted  $Z_q$ ). The total amount of fluid produced at time  $t$  in  $p \in q^{\text{out}}$  is  $M_{pq}Z_q(t)$ . The firing process works as fast as possible. The total amount of fluid which has entered place  $p$  at time  $t$  is denoted  $Z_p(t)$ .

Defining :

$$\mu_{pq} \triangleq M_{pq}, \quad \mu_{qp} \triangleq M_{qp}^{-1}, \quad \tilde{\mu}_{qp} \triangleq \mu_{qp}\rho_{qp},$$

<sup>1</sup>Given a node  $r$  (respectively  $s$ ) (place or transition) is upstream (respectively downstream) a node  $s$  if  $M_{sr} \neq 0$ . We denote  $r^{\text{out}}$  (respectively  $r^{\text{in}}$ ) the set of nodes  $s$  downstream (respectively upstream)  $r$ .

the dynamics of the system are completely defined by :

$$\begin{cases} Z_q(t) &= \min_{p \in q^{\text{in}}} \tilde{\mu}_{qp} Z_p(t - \tau_p), \\ Z_p(t) &= m_p + \sum_{q \in p^{\text{in}}} \mu_{pq} Z_q(t). \end{cases}$$

Eliminating the variable  $Z_p$  we obtain a dynamic programming equation defining  $Z_q$  :

$$Z_q(t) = \min_{p \in q^{\text{in}}} \left[ \tilde{\mu}_{qp} \left( m_p + \sum_{q' \in p^{\text{in}}} \mu_{pq'} Z_{q'}(t - \tau_p) \right) \right]. \quad (1)$$

This equation may be interpreted as a dynamic programming equation for stochastic control problem with a discounted cost.

Under certain condition described by the following theorem this stochastic control problem (1) is undiscounted.

**Theorem 1** *If there exists  $v \in (\mathbb{R}^+)^{\mathcal{Q}}$  such that :*

$$\sum_{q \in p^{\text{out}}} v_q M_{qp} = \sum_{\tilde{q} \in p^{\text{in}}} M_{p\tilde{q}} v_{\tilde{q}}, \quad \forall p \in \mathcal{P}, \quad (2)$$

*equation (1) has the interpretation of an undiscounted stochastic control with Bellman function :*

$$W_q \triangleq \frac{Z_q}{v_q}.$$

In particular the condition of the theorem is fulfilled when :

$$\sum_{q \in p^{\text{out}}} M_{qp} = \sum_{\tilde{q} \in p^{\text{in}}} M_{p\tilde{q}}, \quad \forall p \in \mathcal{P},$$

that is when, for all places, there is an equal number of arcs entering and leaving the place with a uniform routing policy (that is  $\rho_{qp} = 1/|p^{\text{out}}|$  where  $|A|$  of a finite set  $A$  denotes its cardinality). In this case we have  $v = \mathbf{1}$  ( $\mathbf{1}$  is the vector with all entries equal to 1).

Using this remark, we are able to define the throughput of those Petri nets which are the only live and stable Petri nets (the other ones either explode or die after a finite time).

**Theorem 2** *Denoting*

$$P_{qq'}^p = v_q^{-1} \tilde{\mu}_{qp} \mu_{pq'} v_{q'}, \quad v_q^p = m_p \tilde{\mu}_{qp},$$

*the throughput  $\lambda \triangleq \lim_{t \rightarrow \infty} Z_q(t)/t$  of a strongly connected Petri net, satisfying (2), exists, is independent of  $q$  and is solution of the dynamic programming equation :*

$$w_q = \min_{p \in q^{\text{in}}} (v^p - \lambda \tau_p + P^p w)_q, \quad \forall q \in \mathcal{Q}.$$

Using this theorem we can compute the throughput of such Petri net by the Howard algorithm (see [2] for a recent reference) the complexity of which is experimentally almost linear in the number of arcs of the Petri nets.

### 3 Modelling traffic light synchronization by Petri net.

In the sequel we shall propose models of the circulation in an ideal regular town in terms of Petri nets. The purpose of these Petri nets will be to model the coordination between the authorizations given to the cars by the lights. What we will call *cars* in the following will not be real cars but virtual ones which travel like real cars in the town. A real car may decide or not to follow the speed of a virtual car. These virtual cars are useful to study an ideal coordination. The flows of real cars cannot be larger than the virtual ones but they can be smaller. Probabilistic links between virtual and real car will be studied in future work. Here we define only a signal environment for the circulation of real cars.

#### 3.1 Modelling a junction.

The Petri net associated with a junction is given in Figure 2. We denote by  $x_0(t)$  and  $x_1(t)$  the total number of green phases that have happened at each of the two lights until date  $t$ . The green phase lengths of the two lights are denoted respectively  $\tau$  and  $v$ . We suppose that the amount of cars that can cross the junction is proportional to the length of the corresponding green phase with a coefficient that we choose equal to one. We suppose that at each junction a proportion of vehicles equal to  $\alpha$  turn in the only turning direction available at the junction. We denote by  $u_0(t)$  and  $u_1(t)$  the total amount of cars arrived at the junction until time  $t$  and by  $y_0(t)$  and  $y_1(t)$  the total amount of cars which have left the junction until time  $t$ .

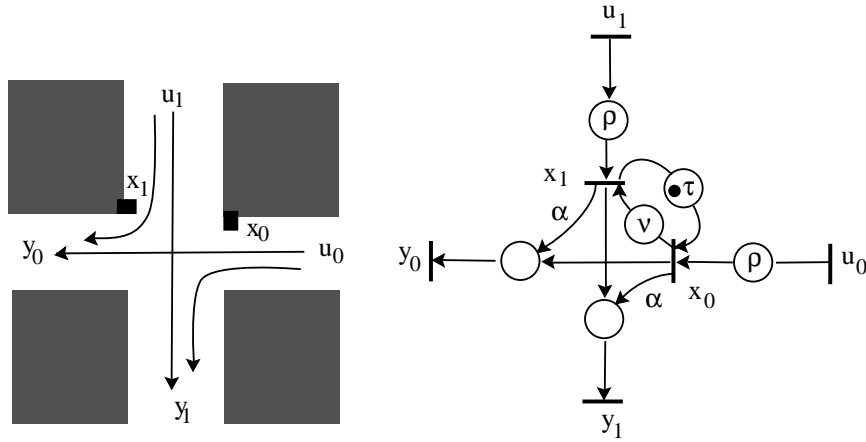


Figure 2: The Petri net of a junction.

The relation between the inputs  $u$  and the output  $y$  is a stochastic dynamic programming equation where the Bellman function is  $x$  :

$$x = a \otimes x \oplus b \otimes u, \quad y = cx, \quad (3)$$



where

$$a = \begin{bmatrix} \epsilon & \gamma\delta^\tau \\ \delta^\nu & \epsilon \end{bmatrix}, \quad b = \begin{bmatrix} \delta^\rho/\nu & \epsilon \\ \epsilon & \delta^\rho/\tau \end{bmatrix}, \quad c = \begin{bmatrix} (1-\alpha)\nu & \alpha\tau \\ \alpha\nu & (1-\alpha)\tau \end{bmatrix},$$

where  $\oplus$  denotes the minplus matrix addition (min elementwise),  $\otimes$  the minplus matrix multiplication (substitution of plus by min and times by plus in the usual matrix product,  $\epsilon = \infty$ ,  $e = 0$ ),  $\delta$  is the unit shift in timing ( $\delta v(t) = v(t-1)$ ) and  $\gamma$  is the unit shift in numbering ( $\gamma v(t) = 1 + v(t)$ ).

With these notations the first equation of (3) means :

$$x_0(t) = \min\{1 + x_1(t-\tau), u(t-\rho)/\nu\}, \quad y_0(t) = (1-\alpha)\nu x_0(t) + \alpha\tau x_1(t).$$

It is important to pay attention that the matrix product in  $y = cx$  is the standard one.

Clearly these dynamic programming equations (3) are neither linear in minplus algebra nor in the standard one. The use of the minplus matrix product is only a convenient and compact way to write vector equations.

### 3.2 Modelling a block of junctions.

Let us consider a regular town, such as the one depicted in Figure 4, composed of squares separated by one way streets (with opposite direction of circulation for successive streets).

To determine the dynamics of this system it is useful to determine first the dynamics of a block composed of four junctions. On the city map it can be observed a regularity characterized by an invariance by horizontal and vertical translation of size 2 blocks.

The dynamics of a block is defined by the Petri net given in Figure 3. The corresponding equations are :

$$\chi_i = a \otimes \chi_i \oplus b \otimes \pi^i \otimes c\chi_{i-1} \oplus b \otimes \pi^{i-1} \otimes E \otimes u_i, \quad y_i = E' \otimes \pi^i \otimes c\chi_i, \quad i = 0, 1, 2, 3,$$

where

$$\chi_i = \begin{bmatrix} x_{2i} \\ x_{2i+1} \end{bmatrix}, \quad \pi^1 = \pi^3 = \begin{bmatrix} e & \epsilon \\ \epsilon & \epsilon \end{bmatrix}, \quad \pi^0 = \pi^2 = \begin{bmatrix} \epsilon & \epsilon \\ \epsilon & e \end{bmatrix}, \quad E = \begin{bmatrix} e \\ e \end{bmatrix},$$

and the computation on the index  $i$  has been done modulo 4.

This is a system with 8 states 4 inputs and 4 outputs system that we can written formally as :

$$x = A \otimes x \oplus B \otimes u, \quad y = Cx,$$

where  $A$  is the  $8 \times 8$  nonlinear operator :

$$A = \begin{bmatrix} a & \epsilon & \epsilon & b\pi^0 c \\ b\pi^1 c & a & \epsilon & \epsilon \\ \epsilon & b\pi^0 c & a & \epsilon \\ \epsilon & \epsilon & b\pi^1 c & a \end{bmatrix}.$$

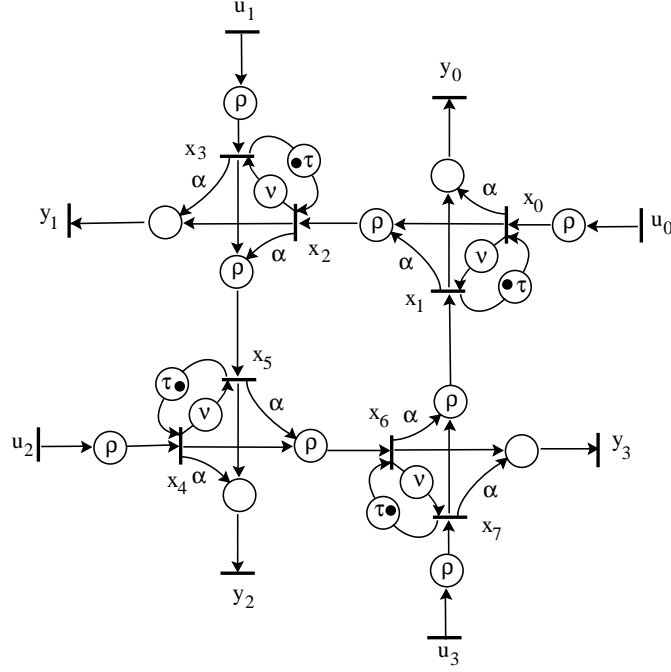


Figure 3: The Petri net of a block of 4 junctions.

The existence of stationary regime is assured as soon as it exists an undiscounted stochastic control interpretation of this dynamic programming equation. For that it is sufficient that

$$(1 - \alpha)v + \alpha\tau = v, \quad (1 - \alpha)\tau + \alpha v = \tau,$$

which implies  $v = \tau$ .

In fact,  $\alpha$ ,  $v$  and  $\tau$  may depend of the junction and in this case the sufficient condition become :

$$\alpha_{q-1}\tau_{q-1} + (1 - \alpha_{q-2})v_{q-2} = v_q, \quad q \text{ even},$$

$$\alpha_{q-3}v_{q-3} + (1 - \alpha_{q-2})\tau_{q-2} = \tau_q, \quad q \text{ odd}.$$

### 3.3 Modelling of a regular town.

A regular town is composed of blocks that we can index with a couple  $(I, J)$  where  $I$  is the west-east (W-E) coordinate of the block and  $J$  the south-north (S-N) coordinate.

Then, the dynamics of a complete town can be written :

$$x_{IJ} = Ax_{IJ} \oplus \mathcal{A}_0x_{I+1,J} \oplus \mathcal{A}_1x_{I,J+1} \oplus \mathcal{A}_2x_{I-1,J} \oplus \mathcal{A}_3x_{I,J-1},$$

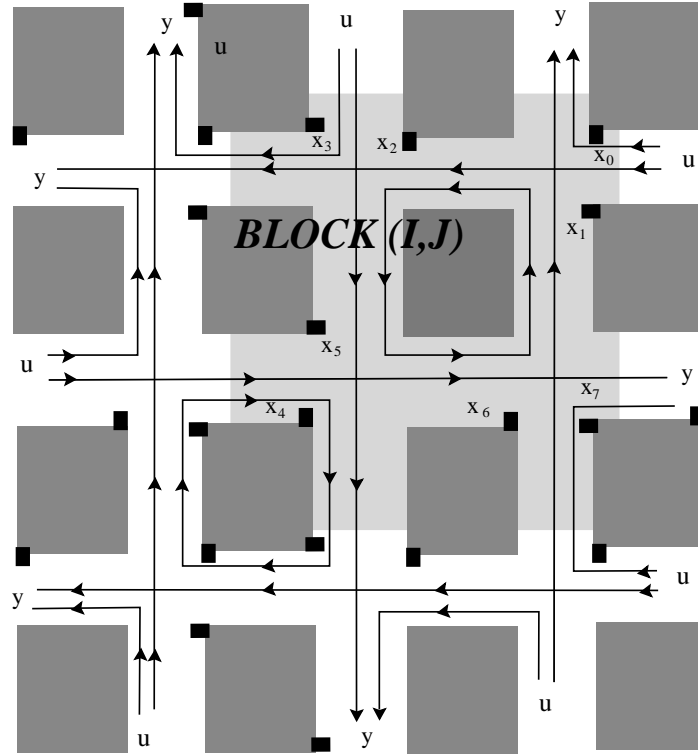


Figure 4: A regular town.

where

$$\mathcal{A}_0 = \begin{bmatrix} \epsilon & b\pi^1 c & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \end{bmatrix}, \quad \mathcal{A}_1 = \begin{bmatrix} \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & b\pi^0 c & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \end{bmatrix},$$

$$\mathcal{A}_2 = \begin{bmatrix} \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & b\pi^1 c \\ \epsilon & \epsilon & \epsilon & \epsilon \end{bmatrix}, \quad \mathcal{A}_3 = \begin{bmatrix} \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon & \epsilon \\ b\pi^0 c & \epsilon & \epsilon & \epsilon \end{bmatrix}.$$

#### 4 Decomposition of the traffic light control problem.

As we have seen in the first section, the previous type of system can be solved efficiently. But for this system we have to optimize some parameters. For example we have to decide the initial markings

in the places corresponding to the streets (the ones containing  $\rho$  in Figure 3), the length of the green and red phases of each light. When there is no saturation, by a good choice of these quantities, we can achieve – a throughput only limited by the full occupation of the junctions – a system of green waves which allows a travel between two points of the town while meeting, at most, one red light.

#### 4.1 Throughput.

It is well known that the throughput of an event graph is the minimum, over all the circuits, of the number of tokens in the circuit divided by the total amount of time that the tokens have to stay in the places of the circuit.

For more general Petri net, for which we have a stochastic control interpretation, it exists [1] an analogous result. In this interpretation the choice of a possible feedback corresponds to the choice of a place upstream each transition in the Petri net. Each feedback defines a Markov chain as we have seen in Theorem 2. This Markov chain has final classes with one invariant measure associated to each final class. These final classes play the role of the event graph circuits, but now we have to average the timings and the tokens with respect to these invariant measures.

More precisely, the throughput  $\lambda$  of a Petri net having the undiscounted stochastic control interpretation is given by the expression :

$$\lambda = \min_{\phi \in \mathcal{F}} \frac{r^\phi \cdot \tilde{m}^\phi}{r^\phi \cdot \tau^\phi} ,$$

where  $\mathcal{F}$  denotes the set of achievable final classes of the Markov chains obtained by choosing only one place upstream each transition,  $r^\phi$  the corresponding invariant measure and

$$\tilde{m}_p \triangleq \begin{cases} m_p & \text{if } p \text{ is a } \tau \text{ or } \nu \text{ place ,} \\ m_p/\nu & \text{if } p \text{ is a } \rho \text{ place arriving in an even transition ,} \\ m_p/\tau & \text{if } p \text{ is a } \rho \text{ place arriving in an odd transition .} \end{cases}$$

Among all the possible final classes there are the ones with only the two arcs representing the two light phases of a junction with a throughput equal to  $1/(\tau + \nu)$ . If we increase the number of tokens in the places corresponding to the street, above some threshold, the optimal final classes will be the ones associated with lights. If we have not put enough tokens in the street places, the effective light cycle can be slowed down (two lights of the same junction may stay red simultaneously).

#### 4.2 Travel time.

We have to design the system in such a way that the throughput be maximal (collective objective). There are many ways to achieve this objective. It is sufficient for that to have, everywhere and always, cars waiting at a light. This is achieved easily by adding virtual cars in the streets. But we may want to minimize the travel times of each virtual car (individual objectives).

The maximal flow by cycle at a light, let us say at an E-W street, is proportional to the amount of green by unit of time  $k\nu/(\nu + \tau)$  (but we have choosen the units such that  $k/(\nu + \tau) = 1$ ). Then

because the traveling time at maximal speed in a street is  $\rho$  the Little formula tells us the number of cars  $m_p$  needed in the street too achieve the wanted flow is :

$$\rho v = m_p .$$

With this number of cars there is no waiting time, that is, there are green waves with the maximal authorized speed.

An interpretation in manufacturing context would be the following :  $m_p = \rho v$  is the minimal number of pallets necessary in an E-W street to achieve the optimal speed given by the slowest machines which are here the junctions seen as machines serving successively E-W and N-S streets.

Clearly for N-S street we have  $\rho \tau = m_p$ .

### 4.3 Orthogonal system of green waves.

The green wave problem can be seen as compatibility conditions between systems of equations. These compatibility conditions will determine the phase difference between the lights and the cycle time of each light.

It is easier to consider a continuous version of the problem. For that let us denote by  $c(t, x, y)$  the color of a  $(x, y)$  point of a town at time  $t$  (each point is supposed to have a light and the light colours belong to a subset of the real numbers). Let us suppose that there is a propagation of the green color with speed  $v(x, y)$  along the  $x$  axis; then  $c$  would satisfy the equation :

$$\partial_t c + v \partial_x c = 0 . \quad (4)$$

Moreover if we suppose that there is another green wave along the  $y$  axis with speed  $w(x, y)$  then  $c$  would also satisfy the equation :

$$\partial_t c + w \partial_y c = 0 . \quad (5)$$

Clearly the system of equations (4) and (5) has not always a non trivial solution. In order to admit a solution  $v$  and  $w$  must satisfy compatibility conditions. This kind of question is well studied. A good reference is [5].

**Theorem 3** *The equations (4) and (5) are compatible iff there exists a potential  $\psi(x, y)$  such that :*

$$\begin{bmatrix} 1/v \\ 1/w \end{bmatrix} = \text{grad}(\psi) . \quad (6)$$

**Proof:** NECESSARY CONDITION. Let us differentiate Eq.4 (divided by  $v$ ) with respect to  $y$  and Eq.5 (divided by  $w$ ) with respect to  $x$ , we obtain

$$(1/v) \partial_{ty} c + \partial_{xy} c + \partial_t c \partial_y (1/v) = 0, \quad (1/w) \partial_{tx} c + \partial_{xy} c + \partial_t c \partial_x (1/w) = 0 .$$

Moreover differentiating Eq.(4) (divided by  $v$ ) and Eq.(5) (divided by  $w$ ) with respect to  $t$ , we obtain

$$\partial_{xt} c + (1/v) \partial_{tt} c = 0, \quad \partial_{yt} c + (1/w) \partial_{tt} c = 0 .$$

It follows that

$$\partial_y(1/v) = \partial_x(1/w) ,$$

which implies (6).

SUFFICIENT CONDITION. If (6) is true,  $c(t, x, y) \triangleq \phi(t - \psi(x, y))$  is solution of the two equations for all  $\phi$  differentiable and therefore the two equations are compatible.  $\square$

The interpretation of this result is easy. A given stationary traffic light plan  $\psi$  defines speeds in the  $x$  and  $y$  direction by (6) at which one sees the light always green. These speeds are not necessarily constant in  $x$  and  $y$ .

From these considerations, if we don't take into account of S-N and W-E streets, the phase difference of the lights in N-S, E-W streets can be chosen to have a fixed speed green wave along all these streets. Clearly we can also realize the green wave for the (S-N, W-E) streets. The coordination between the (S-N, E-W) and (N-S, W-E) green waves impose the length of the time cycle of the lights.

#### 4.4 Light cycle length.

We have taken in the previous section a system of two orthogonal green waves. This can be generalized to two non orthogonal fields of speeds. But when the two fields becomes dependent it does not exist anymore compatibility conditions of the two partial differential equations. But it is possible that the solution of two equations coincide at some point of a mesh. Let us analyze this on the simplest possible situation.

Let us consider a two directions street. It is easy to see that there is no nontrivial solution to :

$$\partial_t c + v \partial_x c = 0, \quad \partial_t c - v \partial_x c = 0 .$$

But consider a mesh of lights of coordinates  $\{x_i\}$  and let us denote  $\rho_i^+$  (respectively  $\rho_i^-$ ) the time for a car to go from  $x_i$  to  $x_{i+1}$  (respectively from  $x_i$  to  $x_{i-1}$ ) and let us denote by  $T_i$  the  $i$ -th light cycle length. As soon as

$$\exists k, k' \in \mathbb{N} : kT_i = \rho_i^+ + \rho_{i+1}^-; \quad k'T_i = \rho_i^- + \rho_{i-1}^+ ,$$

there exists a green wave in the both directions. In the particular cases of a regular mesh and a constant speed we obtain  $kT = 2\rho$ . In practice, the smallest  $k$  ( $k = 1$ ) is acceptable (speed 26 km/h square of side length 130 m gives for  $k = 1$ ,  $T = 36$  s ).

For a 2D regular town we arrive at the same conclusion by considering two circuits having a difference equal to the circuit around a block. Supposing the existence of green waves we can show that  $kT = 2\rho$  by computing the difference of the time length to cover the two circuits.

Moreover, to have a cycle length long enough in practice, we have to choose  $k = 1$  and the light cycle length is determined and is equal to  $2\rho$ .

With this light cycle length the four systems of green waves E-W, W-E, S-N and N-S are compatible. In this town we can go from any point to any other point meeting at most one red light.

### 5 Bahía Blanca example.

The previous discussion has been inspired by the Bahía Blanca town which is a city of Argentina with a very regular design. Clearly this town is enough regular to apply the previous results. The

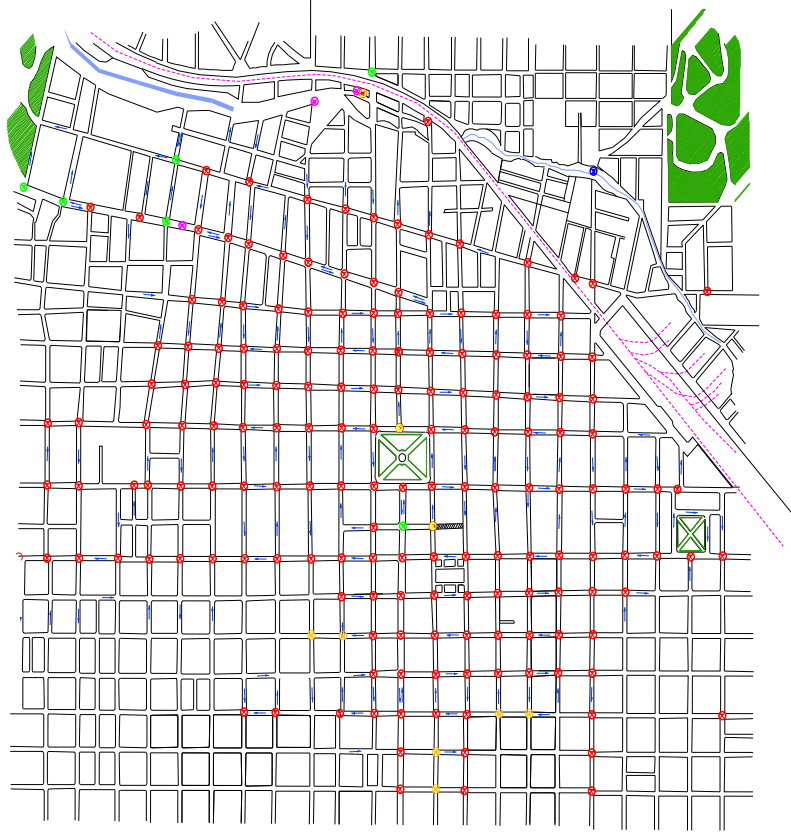


Figure 5: The Bahía Blanca map.

flow of real cars in the street are always changing; therefore we propose to adapt the proportion of green and red lengths according to the real flow without changing the light cycle length and the difference of phase between the lights which can be seen as a coordination between all the lights.

We summarize our proposition of light plan.

1. Determine a reasonable speed  $v$  for the green wave in the town. It must be compatible with a light cycle length equal to  $T = 2\rho$ , with  $\rho = d/v$  where  $d$  is the average length of a square of the town.
2. Using this ideal time  $\rho$  to cover a square length compute the initial starting time of each light cycle. Practically, this means that two successive lights in the same street must have a difference of phase of  $\rho$ .

3. At each junction adapt the proportion of green and red length according to the flow in each direction. If  $\phi$  and  $\psi$  denote the two average flows arriving at the junction and  $\tau$  and  $T - \tau$  the length of the corresponding green phase we must have

$$\phi/(\phi + \psi) = \tau/T .$$

With such a policy, when there is no saturation the average speed of circulation is everywhere approximately  $v$ .

This model is valid only when there is no saturation.

The policy proposed here implies an observation of the flow of cars at all the junctions. This observation is expensive. Traffic assignment techniques, describe for example in [6], could be useful to interpolate the traffic observed at some points and to avoid the installation of some sensors.

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